Period and amplitude

In this lesson, we'll take a more in-depth look at the behavior of the trig functions.

Sine

Notice the following about the behavior of $\sin\theta$:

- The value of $\sin\theta$ increases from 0 to 1 as θ goes from 0 to $\pi/2$.
- The value of $\sin\theta$ decreases from 1 to 0 as θ goes from $\pi/2$ to π.
- The value of $\sin\theta$ decreases from 0 to -1 as θ goes from π to $3\pi/2$.
- The value of $\sin\theta$ increases from -1 to 0 as θ goes from $3\pi/2$ to 2π.

We see this in the graph of $\sin\theta$, and this same behavior repeats over every interval of 2π because sine is periodic, which means the function behaves the same way over $[2\pi,4\pi]$ as it did over $[0,2\pi]$, and behaves the same way over $[4\pi,6\pi]$ as it did over $[2\pi,4\pi]$. And this pattern repeats infinitely in both the positive and negative direction.

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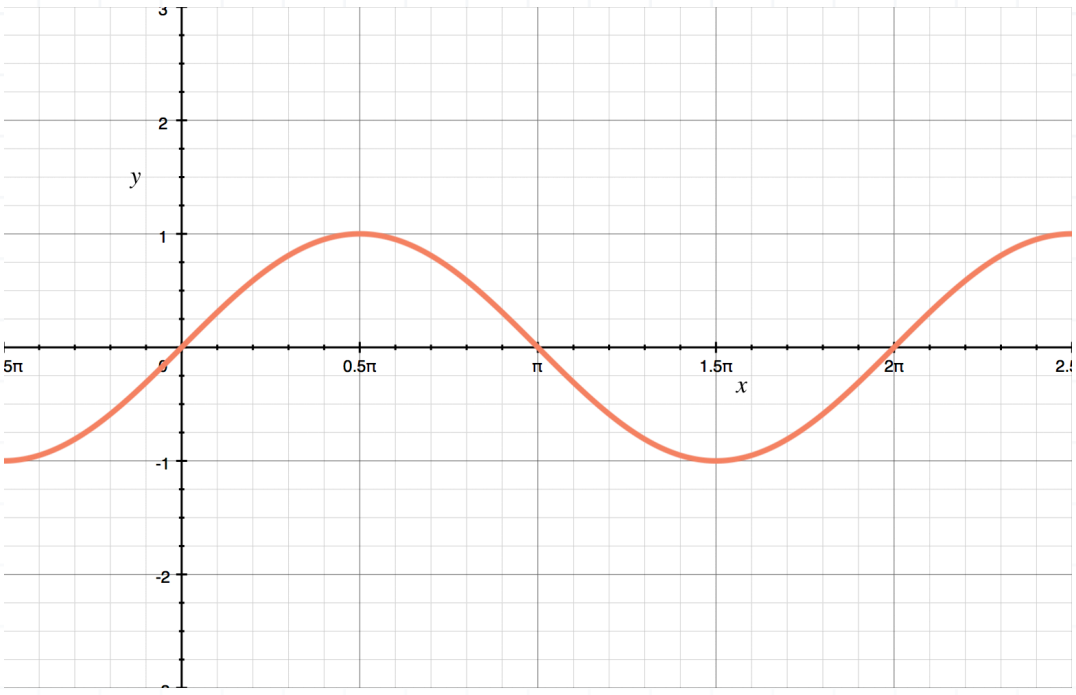

Cosine

Notice the following about the behavior of $\cos\theta$:

- The value of $\cos\theta$ decreases from 1 to 0 as θ goes from 0 to $\pi/2$.
- The value of $\cos\theta$ decreases from 0 to -1 as θ goes from $\pi/2$ to π.
- The value of $\cos\theta$ increases from -1 to 0 as θ goes from π to $3\pi/2$.
- The value of $\cos\theta$ increases from 0 to 1 as θ goes from $3\pi/2$ to 2π.

We see this in the graph of $\cos\theta$, and this same behavior repeats over every interval of 2π because cosine is periodic.

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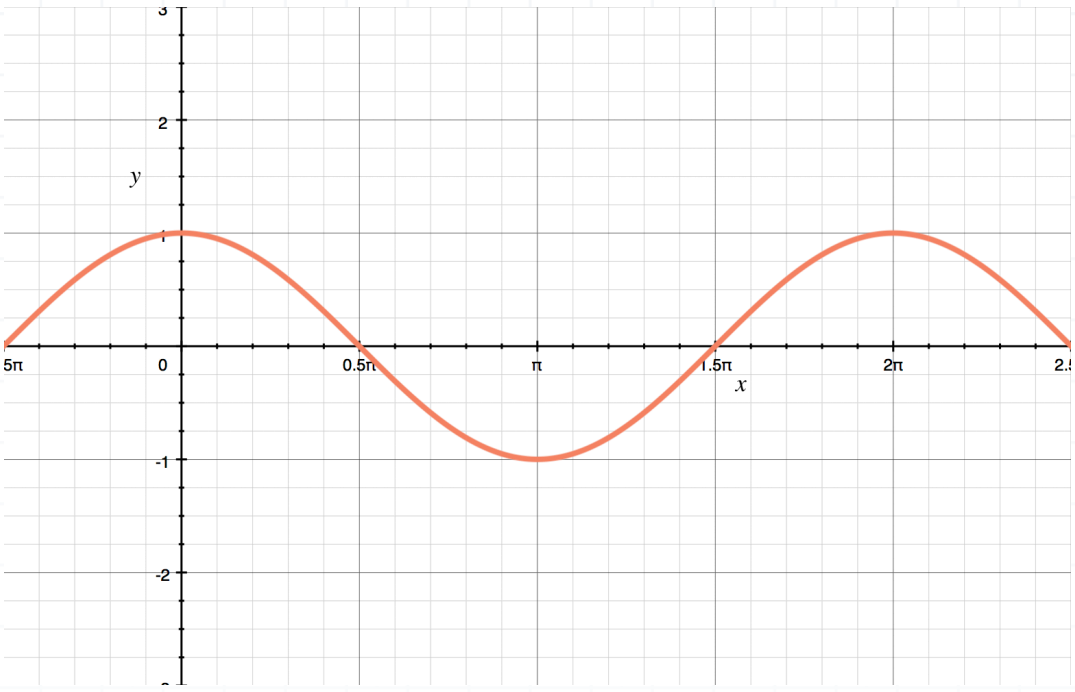

Since the value of $\sin\theta$ oscillates back and forth over the range $[-1,1]$, $\sin\theta$ has amplitude 1, and the same is true of $\cos\theta$.

Given $a\sin\theta$ or $a\cos\theta$ instead of just $\sin\theta$ or $\cos\theta$, where a is a nonzero real number, the value of the sine and cosine functions oscillate between $-|a|$ and $|a|$ (instead of between -1 and 1), so its amplitude is $|a|$.

Example

Find the amplitude of $3\sin\theta$, $3\cos\theta$, $-5\sin\theta$, and $-5\sin\theta$.

For the functions $3\sin\theta$ and $3\cos\theta$, the value of a is 3, so their amplitude is $|3| = 3$. For the functions $-5\sin\theta$ and $-5\cos\theta$, we have $a = -5$, so their amplitude is $|-5| = 5$.

Next, let's take a look at functions of the form

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$$a\sin(b\theta)$$

$$a\cos(b\theta)$$

for some nonzero real numbers a and b. If b is positive, then these functions have an amplitude $|a|$ and period $2\pi/b$.

Example

Find the amplitude and period of $4\sin(6\pi)$.

Here, $a = 4$ and $b = 6$, so the amplitude is $|4| = 4$ and the period is

$$\frac{2\pi}{b} = \frac{2\pi}{|6|} = \frac{2\pi}{6} = \frac{\pi}{3}$$

So what do we do when b is negative? Well consider $2\sin(-5\theta)$ and $2\cos(-5\theta)$. The odd identity for sine tells us that

$$\sin(-5\theta) = -\sin(5\theta)$$

Therefore, the function $2\sin(-5\theta)$ can be written as $-2\sin(5\theta)$.

The even identity for cosine tells us that

$$\cos(-5\theta) = \cos(5\theta)$$

Therefore, the function $2\cos(-5\theta)$ can be written as $2\cos(5\theta)$.

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So we can always express $a\sin(b\theta)$ or $a\cos(b\theta)$ in such a way that b is positive, and we'll do that from this point on.

Example

Express the functions $7\sin(-3\theta)$ and $7\cos(-3\theta)$ in the form $a\sin(b\theta)$ and $a\cos(b\theta)$ where a and b are real numbers and b is positive, and state their amplitudes and periods.

Since $\sin(-3\theta) = -\sin(3\theta)$ and $\cos(-3\theta) = \cos(3\theta)$, we can express the functions $7\sin(-3\theta)$ and $7\cos(-3\theta)$ as $-7\sin(3\theta)$ and $7\cos(3\theta)$, respectively.

The function $-7\sin(3\theta)$ has $a = -7$ and $b = 3$, so its amplitude is $|-7| = 7$ and its period is $2\pi/3$. The function $7\cos(3\theta)$ has $a = 7$ and $b = 3$, so it too has an amplitude of $|7| = 7$ and a period of $2\pi/3$.

Notice that when $b > 1$, the period of the function is smaller than 2π and therefore more than one full period fits into the interval $[0,2\pi)$. When $b < 1$, the period of the function is larger than 2π and less than one full period fits into the interval $[0,2\pi)$.

Secant

Notice the following about the behavior of $\sec\theta$:

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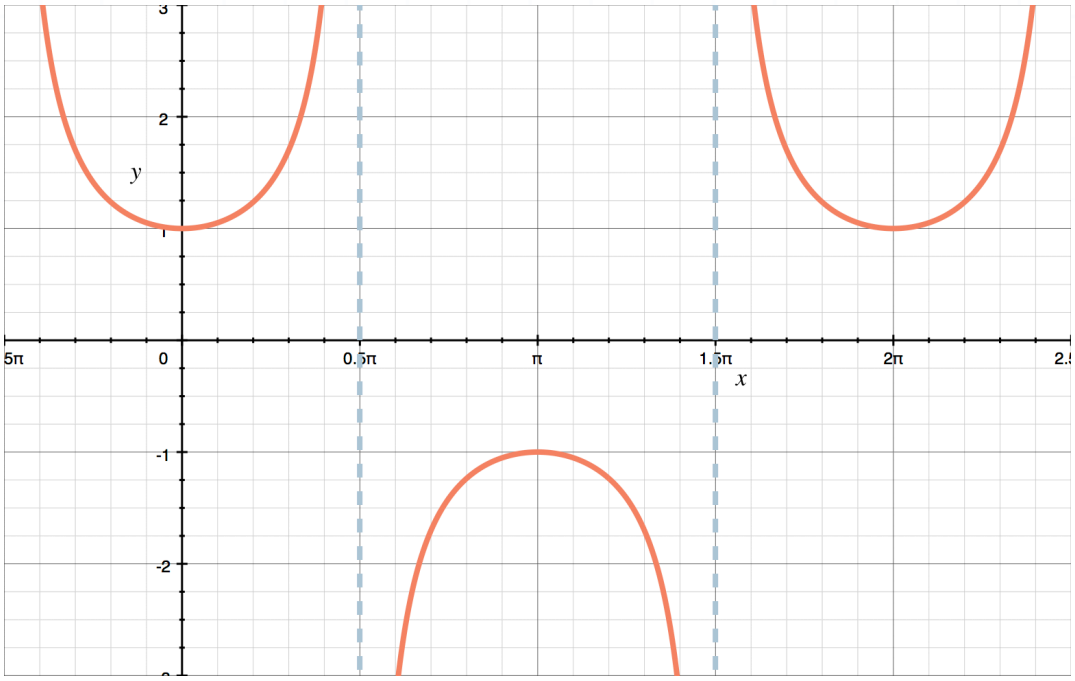
- The value of $\sec\theta$ increases from 1 to ∞ in the interval $[0,\pi/2)$, because the value of $\cos\theta$ decreases from 1 to 0 in that interval.
- The value of $\sec\theta$ increases from $-\infty$ to -1 in the interval $(\pi/2,\pi]$, because the value of $\cos\theta$ decreases from 0 to -1 in that interval.
- The value of $\sec\theta$ decreases from -1 to $-\infty$ in the interval $[\pi,3\pi/2)$, because the value of $\cos\theta$ increases from -1 to 0 in that interval.
- The value of $\sec\theta$ decreases from ∞ to 1 in the interval $(3\pi/2,2\pi]$, because the value of $\cos\theta$ increases from 0 to 1 in that interval.

We see this in the graph of $\sec\theta$, and this same behavior repeats over every interval of 2π because secant is periodic.



Cosecant

Notice the following about the behavior of $\csc\theta$:

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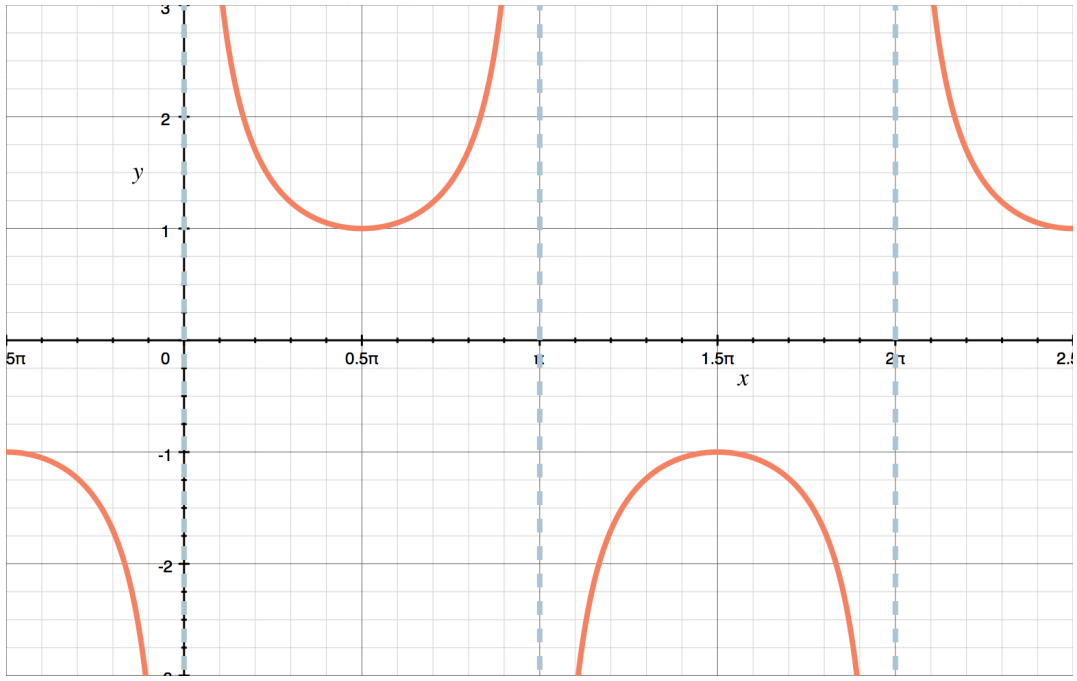
- The value of $\csc\theta$ decreases from ∞ to 1 in the interval $(0,\pi/2]$, because the value of $\sin\theta$ increases from 0 to 1 in that interval.
- The value of $\csc\theta$ increases from 1 to ∞ in the interval $[\pi/2,\pi)$, because the value of $\sin\theta$ decreases from 1 to 0 in that interval.
- The value of $\csc\theta$ increases from $-\infty$ to -1 in the interval $(\pi,3\pi/2]$, because the value of $\sin\theta$ decreases from 0 to -1 in that interval.
- The value of $\csc\theta$ decreases from -1 to $-\infty$ in the interval $[3\pi/2,2\pi)$, because the value of $\sin\theta$ increases from -1 to 0 in that interval.

We see this in the graph of $\csc\theta$, and this same behavior repeats over every interval of 2π because cosecant is periodic.



Tangent

The period of tangent is actually π, not 2π like the first four trig functions. The graph of the tangent function shows its period on the interval $[0,\pi]$.

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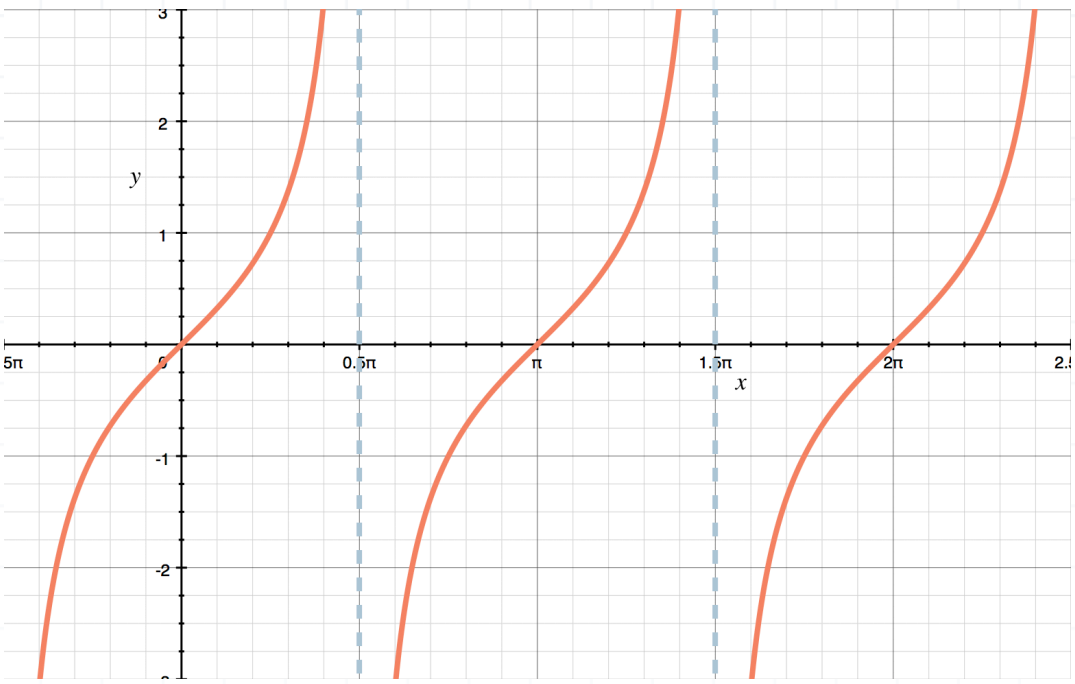
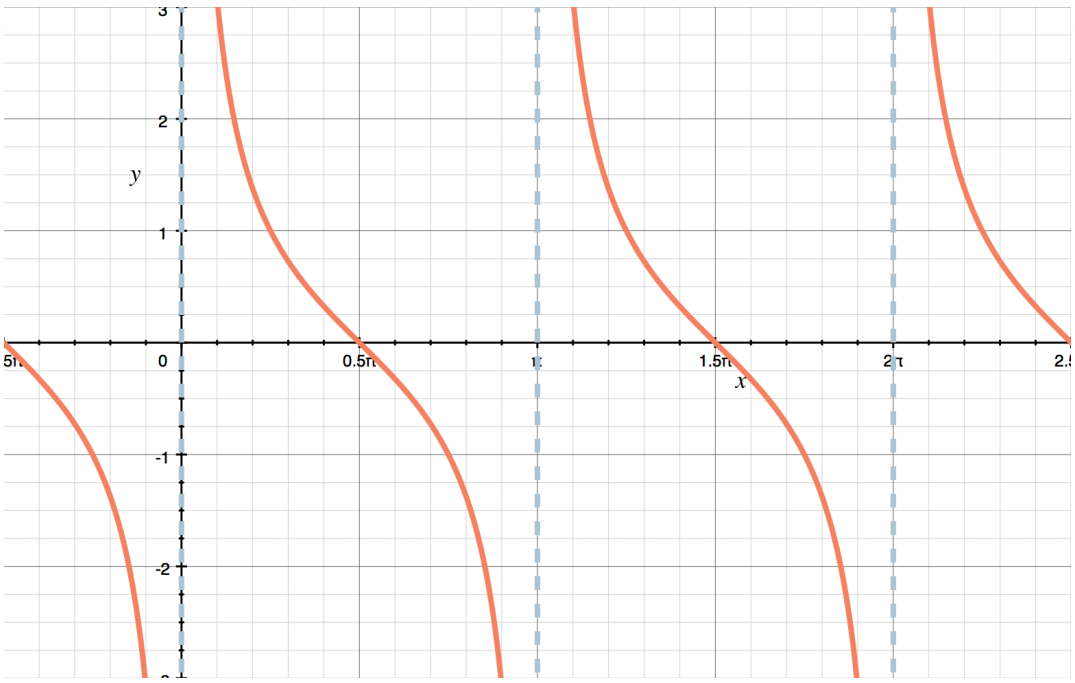


Cotangent

The period of cotangent is like tangent, it's also π. The graph of the cotangent function shows its period on the interval $[0,\pi]$.



Notice that the values of $\sec\theta$, $\csc\theta$, $\tan\theta$, and $\cot\theta$ range from $-\infty$ to ∞, so none of them have a finite range, and they therefore have no amplitude.

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But functions of the form $a\sec(b\theta)$, $a\csc(b\theta)$, $a\tan(b\theta)$, and $a\cot(b\theta)$ are defined for all nonzero real numbers a, b. Just like with sine and cosine, these functions can always be written in a way that makes b positive, which is what we want.

Example

Find the periods of $-9\sec(3\theta/2)$ and $2.5\csc(\theta/6)$.

In the case of $-9\sec(3\theta/2)$, we see that $b = 3/2$. Therefore, the period of this function is

$$\frac{2\pi}{b} = \frac{2\pi}{\frac{3}{2}} = 2\pi\left(\frac{2}{3}\right) = \frac{4\pi}{3}$$

In the case of $2.5\csc(\theta/6)$, the value of b is $1/6$. Therefore, the period of this function is

$$\frac{2\pi}{b} = \frac{2\pi}{\frac{1}{6}} = 2\pi\left(\frac{6}{1}\right) = 12\pi$$

The first four trig functions have a period of 2π, so we can find their period using $2\pi/b$. But tangent and cotangent have a period of π, which means we'll find their periods using π/b.

Example

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Find the periods of $25\tan(\theta/4)$ and $-(1/9)\cot(3\theta)$.

For $25\tan(\theta/4)$, the value of b is $1/4$, so its period is

$$\frac{\pi}{b} = \frac{\pi}{\frac{1}{4}} = \pi\left(\frac{4}{1}\right) = 4\pi$$

For $-(1/9)\cot(3\theta)$, we see that $b = 3$, so its period is

$$\frac{\pi}{b} = \frac{\pi}{3}$$

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